This activity is about using graphical methods to find a function to model the relationship between two sets of data. The function can be used to make predictions and these predictions can then be tested by comparing them to actual results.

Year	Number of cars (000s)
1994	1558
1995	1600
1996	1715
1997	1844
1998	1980
1999	2145
2000	2262
2001	2451
2002	2647
2003	2869
2004	3118
2005	3314
2006	3512
2007	3687

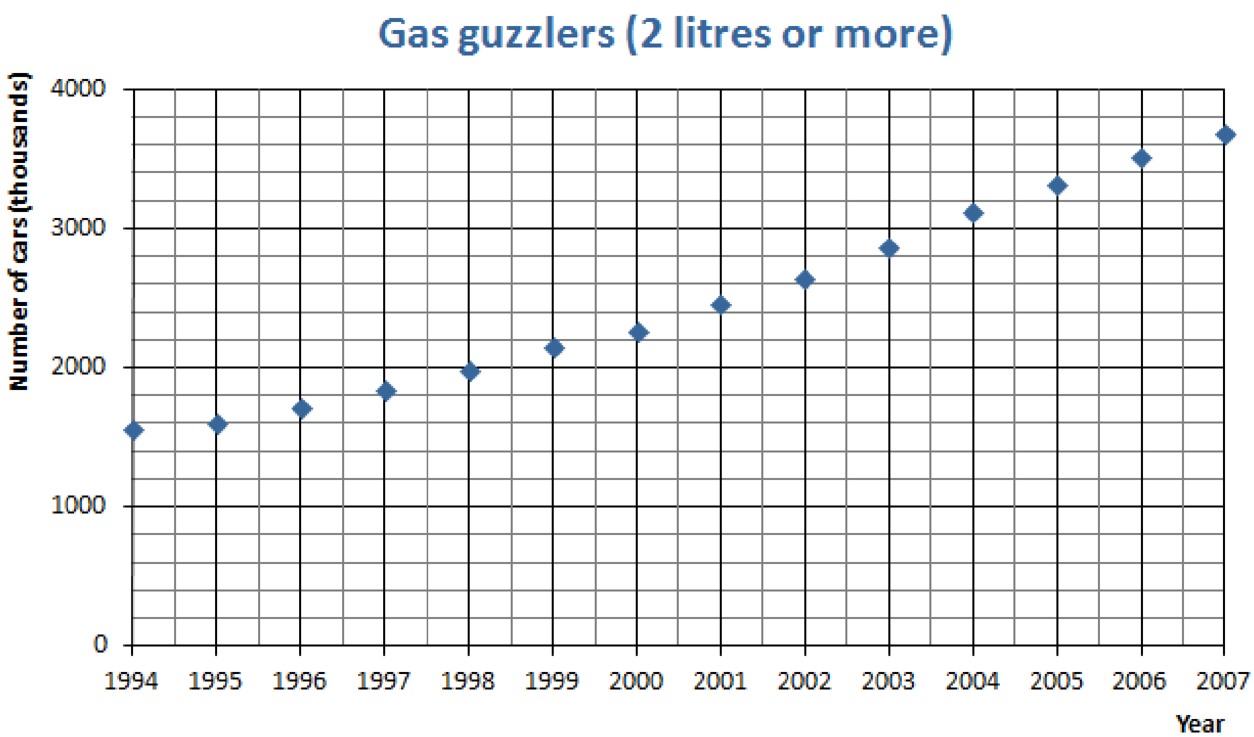
Information sheet

The table gives the number of cars with engine sizes of 2 litres or more that were licensed in Great Britain at the end of each year from 1994 to 2007.

These values are plotted on the graph below.

Think about…

What type(s) of function do you think could be used to model these data?



The values are repeated in the table on the right.

In this table, t represents the number of years since the end of 1994 (so that 0 represents the end of 1994, 1 represents the end of 1995, and so on). N represents the number of thousands of cars with engine sizes of 2 litres or more that were registered.

These data can be modelled by an exponential function of the form $N = N_0 e^{kt}$.

Taking natural logarithms and using the laws

of logarithms:

$$\ln N = \ln N_0 + \ln e^{kt}$$

$$= \ln N_0 + kt \ln e$$

$$\ln N = \ln N_0 + kt$$

Compare this with the linear equation

$$y = c + mx$$

This suggests that a graph of ln N (on the y axis) against t (on the x axis) should give a straight line.

t	N	ln N
0	1558	
1	1600	
2	1715	
3	1844	
4	1980	
5	2145	
6	2262	
7	2451	
8	2647	
9	2869	
10	3118	
11	3314	
12	3512	
13	3687	

Assuming this is so, then the gradient will give the value of k and the intercept on the y axis will give the value of ln N_0.

To answer

1 Complete the ln N column in the table.

2 Draw a graph of ln N against t.

3 Draw the line of best fit.

4 Use the line of best fit to complete the following:

The gradient gives **k =** ……………………………

The intercept on the ln N axis gives: **ln N_0 =** ……………………………

N_0 = ……………………………

The exponential model is **N =** ……………………………

Think about …

How can you check how well your model fits the data?

To answer

5 Use one or both of the following methods to compare values suggested by the model with the data used to find it.

a Find percentage errors using:

% error

$$= \frac{\text{predicted value} - \text{actual value}}{\text{actual value}} \times 100$$

b Plot a graph showing both the values suggested by the model and the actual values.

t	Data	Model	% error
0	1558		
1	1600		
2	1715		
3	1844		
4	1980		
5	2145		
6	2262		
7	2451		
8	2647		
9	2869		
10	3118		
11	3314		
12	3512		
13	3687		

6 What does the model predict for the years 2008 and 2009?

7 The actual values for 2008 and 2009 were 3731 thousand and 3768 thousand respectively. Comment on the accuracy of your predictions.

8 Draw a new graph for the years 2000 to 2009.
Find a new function to model these data.

Reflect on your work

- Why is the percentage error a better measure of the accuracy of a model than the difference between the actual value and the value predicted by the model?
- What is indicated by a negative percentage error?
- Is your model valid for all values of t ?